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Convection in a porous medium with inclined temperature gradient

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Abstract—The stability of convection in a horizontal porous layer, subjected to an inclined temperature gradient of finite magnitude, and confined between perfectly conducting planes, is investigated by means of linear stability analysis. It is shown that the instability appears in the form of stationary longitudinal rolls (with axes aligned in the direction of the horizontal component of temperature gradient) superimposed on the basic flow. As the horizontal Rayleigh number increases, the critical vertical Rayleigh number also increases and there is a series of transitions to higher order modes, corresponding to multiple layers of rolls.

1. INTRODUCTION

WHETHER or not a convective flow is stable is of practical importance because when instability occurs the rate of transfer of heat is increased. In this paper we study the stability of the steady convective flow (of Hadley type) which is set up by the horizontal component of a temperature gradient in a shallow horizontal layer of a saturated porous medium, the instability resulting from the presence of the vertical component of the temperature gradient. Weber [1] studied this problem, but his analysis was limited to the case of small horizontal temperature gradients. In this paper we remove the restriction that the horizontal temperature gradient be small. Standard linear analysis is employed. A Galerkin approximation is made in solving the resulting eigenvalue equation. The advantage of this approach, compared with a routine numerical investigation, is that a large parameter space can be dealt with in an economic manner.

2. GOVERNING EQUATIONS AND BASIC SOLUTION

We will adopt notation which is essentially the same as that of Weber [1]. With the y^*-axis taken vertically upwards, the fluid is assumed to lie between the impermeable planes $y^* = \pm h/2$, so h is the layer depth. (Asterisks denote dimensional variables.) The temperature boundary conditions are

$$T^* = T_0^* \mp \Delta T^*/2 - \beta^* x^* \quad \text{at} \quad y^* = \pm h/2$$

so that, at a given value of x^*, the lower plane is hotter than the upper plane by an amount ΔT^*, and it is assumed that β^* (the horizontal temperature gradient) is a positive constant. We introduce dimensionless variables by choosing as scales h for length, $(c_p\rho)_m h^2/\lambda_m$ for time, κ_m/h for velocity, ΔT^* for temperature and $\rho_0 \nu \kappa_m/K$ for pressure. Assuming that Darcy's law is valid and making the Oberbeck–Boussinesq approximation, we can write the governing differential equations in the dimensionless form

$$\nabla p + \mathbf{v} - RT\mathbf{j} = 0 \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

$$\partial T/\partial t + \mathbf{v} \cdot \nabla T - \nabla^2 T = 0 \tag{3}$$

where the Rayleigh number $R = g\gamma(\Delta T^*)Kh/\kappa_m \nu$. (For Darcy's law to be valid, there is an implied restriction on the magnitude of β^*. We shall return to this point later.)

The thermal boundary conditions are now

$$T = \mp 1/2 - \beta x \quad \text{at} \quad y = \pm 1/2 \tag{4}$$

where $\beta = \beta^* h/\Delta T^*$.

The system (1)–(4) permits the steady solution

$$u = U(y),\ v = w = 0,\ T = T(y) - \beta x,\ p = P(x, y) \tag{5}$$

provided that

$$DU(y) = \beta R \tag{6}$$

$$D^2 T(y) = -\beta U(y) \tag{7}$$

(where $D = d/dy$)

$$T(\pm \tfrac{1}{2}) = 0. \tag{8}$$

We shall assume that there is no net mass flux, so

$$\langle U(y) \rangle = 0 \tag{9}$$

(where $\langle (\cdot) \rangle = \int_{-1/2}^{1/2} (\cdot)\, dy$).

The solution of system (6)–(9) is

$$U(y) = \beta R y \tag{10}$$

$$T(y) = -y + \tfrac{1}{24}\beta^2 R(y - 4y^3). \tag{11}$$

NOMENCLATURE

A_{ij} matrix element
A, B, C, D constants in equation (18)
c form drag constant
c_p specific heat at constant pressure
d characteristic grain diameter
D differential operator, d/dy
g gravitational acceleration
h depth of porous medium
i, j, k unit vectors
K permeability
k, m dimensionless wave number in the x- and z-directions
M constant defined in equation (23)
P Prandtl number, ν/κ_m
p, q quantities defined in equation (23)
R vertical Rayleigh number, $g\gamma(\Delta T^*)Kh/\kappa_m \nu$
t dimensionless time
v seepage velocity vector, (u, v, w)
R_H horizontal Rayleigh number, $\beta R = g\gamma\beta^* Kh^2/\kappa_m \nu$
$U(y), T(y), P(x, y)$ dimensionless basic velocity, temperature and pressure
x, y, z dimensionless Cartesian coordinates
T dimensionless temperature
T_0^* standard temperature
ΔT^* temperature difference between lower and upper boundaries.

Greek symbols
α dimensionless overall wave number
β dimensionless horizontal temperature gradient, $\beta^* h/\Delta T^*$
γ coefficient of volume expansion
θ dimensionless temperature perturbation
κ thermal diffusivity, $\lambda_m/(c_p \rho)_f$
λ_m thermal conductivity
ν kinematic viscosity
ρ density
ρ_0 standard density
σ frequency for disturbances.

Subscripts
c critical
f fluid
m solid–fluid mixture.

Superscripts
* dimensional quantities
$\hat{}$ perturbation quantities
$\tilde{}$ scaled quantities defined by equation (23).

3. PERTURBATION ANALYSIS

We now let

$$\mathbf{v} = U(y)\mathbf{i} + \hat{\mathbf{v}}(x, y, z, t)$$
$$T = T(y) - \beta x + \hat{\theta}(x, y, t)$$
$$p = P(x, y) + \hat{p}(x, y, z, t) \qquad (12)$$

and assume that perturbation quantities (those with carets) are small. We substitute into equations (1)–(4), linearize by neglecting products of small quantities and thus obtain

$$\nabla \hat{p} + \hat{\mathbf{v}} - R\hat{\theta}\mathbf{j} = 0$$
$$\nabla \cdot \hat{\mathbf{v}} = 0$$
$$\partial\hat{\theta}/\partial t + U\partial\hat{\theta}/\partial x + \hat{v}DT - \beta\hat{u} - \nabla^2\hat{\theta} = 0. \qquad (13)$$

We then make a normal mode expansion, letting

$$[\hat{u}, \hat{v}, \hat{w}, \hat{\theta}, \hat{p}] = [u(y), v(y), w(y), \theta(y), p(y)] \times \exp\{i(kx + mz - \sigma t)\}.$$

Substituting in equation (13), eliminating $u(y)$, $w(y)$ and $p(y)$, and writing

$$\alpha^2 = k^2 + m^2$$

we obtain the pair of ordinary differential equations

$$(D^2 - \alpha^2)v + R\alpha^2\theta = 0 \qquad (14)$$

$$(D^2 - \alpha^2 - ikU + i\sigma)\theta + i\beta(k/\alpha^2)Dv - vDT = 0. \qquad (15)$$

These two equations must be solved subject to the boundary conditions (appropriate for conducting impermeable boundaries)

$$v = \theta = 0 \quad \text{at} \quad y = \pm 1/2. \qquad (16)$$

Equations (14)–(16) constitute an eigenvalue problem in which R may be regarded as the eigenvalue.

4. SOLUTION OF THE EIGENVALUE PROBLEM

We obtain an approximate solution of the eigenvalue problem using the Galerkin method. Even at low order, this method has been found to yield an excellent approximation in similar problems (see, for example, Finlayson [2]). As a first step we take a second-order approximation. This enables us to deal qualitatively with the various modes of disturbance (odd and even modes, stationary and travelling modes) which must be considered. We discuss the accuracy of the approximation in Section 5. We choose as trial functions trigonometric functions which satisfy the boundary conditions exactly. (They also satisfy the differential equations exactly when $\beta = 0$.) We take

$$v_{2j-1} = \theta_{2j-1} = \cos(2j-1)\pi y$$
$$v_{2j} = \theta_{2j} = \sin 2j\pi y. \quad (17)$$

To start with, we put

$$v = Av_1 + Bv_2, \quad \theta = C\theta_1 + D\theta_2. \quad (18)$$

We substitute these expressions into equations (14) and (15), multiply the first equation by v_1 and v_2 in turn, multiply the second equation by θ_1 and θ_2 in turn, integrate each term from $y = -1/2$ to $1/2$ and perform some integrations by parts utilizing the boundary conditions and then eliminate the constants A, B, C and D from the resulting four homogeneous linear equations, to obtain the eigenvalue equation in the form

$$\det(A_{ij}) = 0 \quad (19)$$

where, for $i, j = 1, 2$

$$A_{ij} = \langle Dv_i Dv_j + \alpha^2 v_i v_j \rangle$$
$$A_{i+2,j} = -R\alpha^2 \langle \theta_i v_j \rangle$$
$$A_{i,j+2} = \langle v_i \theta_j DT - ik\beta\alpha^{-2}\theta_i Dv_j \rangle$$
$$A_{i+2,j+2} = \langle D\theta_i D\theta_j + (\alpha^2 - i\sigma + ikU)\theta_i\theta_j \rangle.$$

All these integrals are easily evaluated, and after some row and column multiplications equation (19) yields

$$\begin{vmatrix} \pi^2+\alpha^2 & 0 & R\alpha^2 & 0 \\ 0 & 4\pi^2+\alpha^2 & 0 & R\alpha^2 \\ 1-\beta^2 R/4\pi^2 & (8/3)ik\beta/\alpha^2 & \pi^2+\alpha^2-i\sigma & (16/9\pi^2)ik\beta R \\ -(8/3)ik\beta/\alpha^2 & 1-\beta^2 R/16\pi^2 & (16/9\pi^2)ik\beta R & 4\pi^2+\alpha^2-i\sigma \end{vmatrix} = 0. \quad (20)$$

Since we require solutions that are bounded as x and y become large, the wave numbers k and α must be real. At neutral stability, σ is also real. Then, expanding out the determinant and taking the real and imaginary parts of equation (20) we obtain the pair of simultaneous equations

$$(\tilde{R} - \tilde{R}_H^2 - p^2/4\tilde{\alpha}^2)(\tilde{R} - \tilde{R}_H^2/4 - q^2/4\tilde{\alpha}^2)$$
$$+ M\hat{k}^2\tilde{R}_H^2\tilde{\alpha}^{-2}(pq + 9/4) - pq\tilde{\sigma}^2/16\tilde{\alpha}^4 = 0 \quad (21)$$
$$\tilde{\sigma}\{\tilde{R} - \tilde{R}_H^2(p+4q)/(4p+4q) - pq/4\tilde{\alpha}^2\} = 0. \quad (22)$$

Here

$$\tilde{R} = R/4\pi^2, \quad \tilde{R}_H = R_H/4\pi^2 = \beta R/4\pi^2$$
$$\tilde{\alpha} = \alpha/\pi, \quad \hat{k} = k/\alpha, \quad \tilde{\sigma} = \sigma/\pi^2$$
$$p = 1 + \tilde{\alpha}^2, \quad q = 4 + \tilde{\alpha}^2, \quad M = 256/81\pi^2. \quad (23)$$

The quantities with tildes and the symbols p, q and M have been introduced as abbreviations. The symbol $\hat{k} = k/(k^2+m^2)^{1/2}$ takes the value unity for transverse modes ($m = 0$) and the value zero for longitudinal modes ($k = 0$). We have introduced a 'horizontal Rayleigh number' R_H defined by

$$R_H = \beta R = g\gamma\beta^* K h^2/\kappa_m \nu \quad (24)$$

and from now on we shall refer to R as the 'vertical Rayleigh number'.

It is now appropriate to consider the various types of modes of disturbance.

4.1. *Stationary longitudinal modes* ($\tilde{\sigma} = 0$, $\hat{k} = 0$)

Equation (22) is satisfied, and equation (21) yields the alternatives, either

$$\tilde{R} = \tilde{R}_H^2 + (1+\tilde{\alpha}^2)^2/4\tilde{\alpha}^2 \quad (25)$$

or

$$\tilde{R} = \tilde{R}_H^2/4 + (4+\alpha^2)^2/4\alpha^2. \quad (26)$$

For the first mode, given by equation (25), the minimum value of $\tilde{R}$ as the wave number $\tilde{\alpha}$ varies is attained when $\tilde{\alpha} = 1$, and is given by

$$\tilde{R}_{LS1} = 1 + \tilde{R}_H^2 \quad (27)$$

that is

$$R_{LS1} = 4\pi^2 + R_H^2/4\pi^2. \quad (28)$$

Since $R_H = \beta R$ this agrees with the approximation for small β obtained by Weber [1], namely

$$R = 4\pi^2(1+\beta^2). \quad (29)$$

For the second mode, given by equation (26), the minimum value of $\tilde{R}$ is attained when $\tilde{\alpha} = 2$, and is given by

$$\tilde{R}_{LS2} = 4 + \tilde{R}_H^2/4. \quad (30)$$

We see that $\tilde{R}_{LS2} > \tilde{R}_{LS1}$ for $\tilde{R}_H < 2$, but $\tilde{R}_{LS2} < \tilde{R}_{LS1}$ when $\tilde{R}_H > 2$.

4.2. *Stationary transverse modes* ($\tilde{\sigma} = 0$, $\hat{k} = 1$)

For this case, equation (21) gives

$$(\tilde{R} - \tilde{R}_H^2 - p^2/4\tilde{\alpha}^2)(\tilde{R} - \tilde{R}_H^2/4 - q^2/4\tilde{\alpha}^2)$$
$$+ M\tilde{R}_H^2\tilde{\alpha}^{-2}(pq + 9/4) = 0. \quad (31)$$

For given $\tilde{\alpha}$ and $\tilde{R}_H$, this is a quadratic equation for $\tilde{R}$, which can easily be solved numerically. The results for the critical value $\tilde{R}_{TS1}$ (the minimum as α varies, of the smaller root) are plotted in Fig. 1. It is seen that $\tilde{R}_{LS1}$ is always less than $\tilde{R}_{TS1}$ for $\tilde{R}_H \neq 0$, i.e. the longitudinal mode is more unstable than the transverse mode. It is only for small values of $\tilde{R}_H$ that the second root for $\tilde{R}$ takes a minimum as α varies. The minimum value $\tilde{R}_{TS2}$ is also plotted in Fig. 1. (For larger values of $\tilde{R}_H$, this second root has a maximum (rather than a minimum) as α varies.)

4.3. *Travelling waves modes* ($\tilde{\sigma} \neq 0$)

Now equation (22) requires that

$$\tilde{R} = \tilde{R}_H^2(p+4q)/(4p+4q) + pq/4\tilde{\alpha}^2. \quad (32)$$

This does not depend on the direction of the hori-

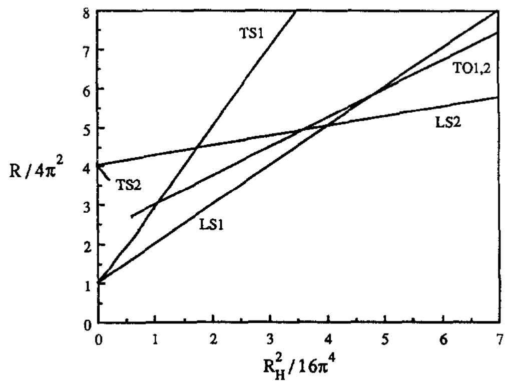

FIG. 1. Plots relating critical values of the vertical Rayleigh number R and the horizontal Rayleigh number R_H for various modes of instability. The modes are designated LS for longitudinal stationary, TS for transverse stationary and TO for transverse oscillatory.

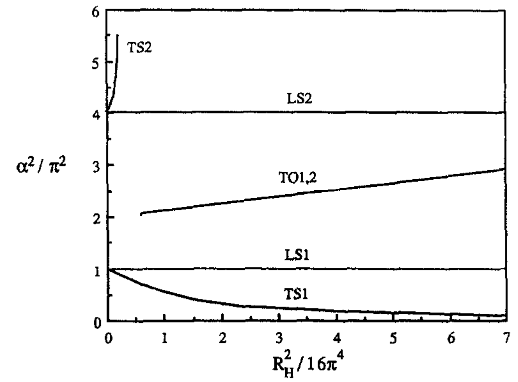

FIG. 2. Plots relating critical values of the non-dimensional horizontal wave number α and the horizontal Rayleigh number R_H, for various modes of instability labelled as in Fig. 1.

zontal wave number vector (as expressed by $\hat{k}$). However, the frequency $\tilde{\sigma}$ does depend on $\hat{k}$. From equation (21), together with equation (32), we find that

$$\tilde{\sigma}^2 = -9[1-\tilde{R}_H^2\tilde{\alpha}^2/(5+2\tilde{\alpha}^2)]^2 + (1024/9)\pi^2\hat{k}^2\tilde{R}_H^2\tilde{\alpha}^2[(1+\tilde{\alpha}^2)^{-1}(4+\tilde{\alpha}^2)^{-1}+4/9]. \quad (33)$$

For longitudinal modes ($\hat{k}=0$) this gives no real value for $\tilde{\sigma}$ (and so there are no unstable longitudinal travelling wave modes) while for transverse modes ($\hat{k}=1$) it is only for $\tilde{R}_H$ exceeding a certain cut-off value that real values of $\tilde{\sigma}$ are possible. For such values of $\tilde{R}_H$ we return to equation (32) and minimize $\tilde{R}$ with respect to $\tilde{\alpha}$, to obtain the critical value $\tilde{R}_{TO1,2}$, the critical vertical Rayleigh number for transverse oscillatory disturbances which involve an oscillation in time between a state with eigenfunction (v_1, θ_1) and a state with eigenfunction (v_2, θ_2). (It is possible to calculate the ratios B/A and D/C for the coefficients in equation (17). In general, these ratios are not purely imaginary, and so the time interval from the first to the second state is not equal to that for the return from the second state to the first.) The values for $\tilde{R}_{TO1,2}$ obtained numerically are also plotted in Fig. 1. It is seen that $\tilde{R}_{TO1,2}$ is always greater than the minimum of $\tilde{R}_{LS1}$ and $\tilde{R}_{LS2}$.

In each case, the corresponding values of the critical horizontal wave number α are plotted in Fig. 2. It is noteworthy that $\tilde{\alpha}_{TS1}$ decreases as $\tilde{R}_H$ increases. As one might expect, $\tilde{\alpha}_{TO1,2}$ takes values intermediate between those of $\tilde{\alpha}_{LS1}$ and $\tilde{\alpha}_{LS2}$.

4.4. *Higher order modes*

The Galerkin process may be repeated with v_1, v_2, θ_1, θ_2 replaced by functions of the form (17) with other values of j. One finds that, as far as stationary modes are concerned, the transverse modes are always more stable than the longitudinal modes. The critical vertical Rayleigh number for the mth longitudinal mode is given by

$$\tilde{R}_{LSm} = m^2 + \tilde{R}_H^2/m^2.$$

Longitudinal oscillatory modes are never unstable. For transverse oscillatory modes the critical vertical Rayleigh number for the interaction between the mth and nth states is given by

$$\tilde{R}_{TOm,n} = (m+n)^2/4 + \tilde{R}_H^2(m^3+n^3)/m^2n^2(m+n).$$

It is easily checked that $\tilde{R}_{TOm,n}$ is always greater than the minimum of the pair R_{LSm} and R_{LSn}. Hence the oscillatory modes are never the favoured ones for the onset of instability. In fact, for R_H in the range

$$m(m-1) \leqslant \tilde{R}_H \leqslant m(m+1)$$

the overall critical vertical Rayleigh number is $\tilde{R}_{LSm}$ which corresponds to the wave number $\tilde{\alpha} = m$. That is, for

$$4m(m-1)\pi^2 \leqslant R_H \leqslant 4m(m+1)\pi^2$$

we have

$$R_c = 4m^2\pi^2 + R_H^2/4m^2\pi^2, \quad \alpha_c = m\pi.$$

5. DISCUSSION

5.1. *The mechanism of the instability*

Looking back through the analysis, we see that the effect of the horizontal temperature gradient on the instability of the longitudinal modes arises through the terms of the form $\langle v\theta DT\rangle$ where T is given by equation (11), so

$$DT = -1 + \tfrac{1}{24}\beta^2R(1-12y^2). \quad (34)$$

With $\langle v\theta\rangle$ positive, the term $-\langle v\theta DT\rangle$ can be interpreted as a rate of transfer of energy into the disturbance by interaction of the perturbation convective motion with the basic temperature gradient, and clearly this has to be positive if instability is to occur. For example

$$-\langle v_1\theta_1 DT\rangle = \frac{1}{2} - \frac{\beta^2R}{8\pi^2} \quad (35)$$

so that a necessary condition for instability is that

$\beta^2 R < 4\pi^2$, that is (since $R_H = \beta R$), $R_H^2/4\pi^2 < R$. (In fact, we know from equation (28) that instability requires that $R_H^2/4\pi^2 < R - 4\pi^2$ for this mode.) We note that the agency causing the instability is essentially a thermal one, so we have a situation which contrasts with the instability of shear flows in which a mechanism involving the transfer of momentum is involved. Also, it is clear that the effect of increasing β is stabilizing because it distorts the basic temperature profile away from the linear one (and ultimately changes the sign of its slope in the centre of the channel). Further, we find that

$$-\langle v_m \theta_m \mathrm{D}T \rangle = \frac{1}{2} - \frac{\beta^2 R}{8\pi^2 m^2}. \tag{36}$$

Thus the stabilizing effect from the distortion of the basic temperature profile decreases as m increases, and this explains the transition to the next higher order mode (as the favoured form of disturbance) when the horizontal temperature gradient increases.

We also note that for the transverse modes there are additional terms like $\langle ik\beta\alpha^{-2}\theta_1 \mathrm{D}v_2 \rangle$ which arise from an interaction involving the convective transfer of heat in the x-direction. These terms are out of phase with the other perturbation energy transport terms. They lead to the possibility of instability involving transverse oscillatory disturbances.

5.2. *The limitations of the model and the analysis*

Here we have been considering an idealized situation, one in which the length-to-depth aspect ratio is sufficiently large so that end effects may be neglected, and hence the basic flow is unidirectional. If, in fact, we have flow in a box with walls at $x^* = \pm L/2$, then the horizontal temperature difference is $\beta^* L$. For the Boussinesq approximation to remain valid, we require that $\gamma\beta^* L \ll 1$. If $\gamma\beta^* h = O(\varepsilon)$ and $R_H = O(1)$, then $gKh/\kappa_m \nu = O(\varepsilon^{-1})$ and so $\gamma\beta^*\kappa_m \nu/gK = O(\varepsilon^2)$. This imposes an upper limit on the size of β^*, but it appears that, at least for the case of liquids, this restriction will not be serious in most situations. We conclude that R_H values of order unity should be obtainable without invalidating the Boussinesq approximation, but clearly the approximation will be invalidated for sufficiently large horizontal temperature gradients.

Further, as β^* increases the magnitude of the basic velocity increases, and sooner or later Darcy's law will no longer be applicable. Weber [1] found a condition that a Reynolds number based on the characteristic basic velocity was sufficiently small, which implies that analysis based on Darcy's laws is valid if

$$R_H < 2Ph/d \tag{37}$$

where P is the Prandtl number and d the characteristic grain diameter. A study of what happens when Darcy's law is replaced by Forchheimer's quadratic drag law indicates that stability results based on Darcy's law are valid if

$$R_H < Ph/cK^{1/2} \tag{38}$$

where c is the form drag constant (the value of which is of the order of 0.5) and K the permeability. Criterion (35) is essentially the same as (34). Insertion of some typical values of the various physical quantities shows that Darcy's law will be valid in nearly all laboratory situations, the exceptions being those involving liquid metals for which the kinematic viscosity is particularly small.

We now turn to the question of the accuracy of our second-order Galerkin approximation. Having determined that it is the longitudinal stationary modes which are the most unstable modes, it is feasible to carry the algebraic treatment one step further, using as trial functions the set $v_1 = \theta_1 = \cos \pi y$, $v_3 = \theta_3 = \cos 3\pi y$ for the lowest even mode and the set $v_2 = \theta_2 = \sin 2\pi y$, $v_4 = \theta_4 = \sin 4\pi y$ for the lowest odd mode. By this means we have found that the second-order approximation is highly accurate so long as $\tilde{R}_H$ has a value of unity or smaller. For example, when $\tilde{R}_H = 1$, the critical value of $\tilde{R}$ is overestimated by about 1% and the critical value of $\tilde{\alpha}$ is underestimated by about 4%.

But as $\tilde{R}_H$ increases, the accuracy of the second-order approximation rapidly decreases. When $\tilde{R}_H = 2$, the value for the switch-over from the first longitudinal stationary mode to the second, the critical value of $\tilde{R}$ is overestimated by 20%. For larger values of $\tilde{R}_H$ a large number of trial functions will be necessary to get an accurate approximation. Indeed, at very large values of $\tilde{R}_H$ the Galerkin approximation will not be feasible, even when a computer is employed, and some sort of asymptotic analysis will be needed. In view of the above limitations of the model itself, it is doubtful whether the effort of performing this additional work is justified.

At present there are no experimental observations available, even for the case of small values of $\tilde{R}_H$. When some experiments have been performed, it may be desirable to use a computer to perform computations for intermediate values of $\tilde{R}_H$. In the meantime, our approximate results provide upper bounds on the critical vertical Rayleigh number.

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CONVECTION DANS UN MILIEU POREUX AVEC UN GRADIENT DE TEMPERATURE INCLINE

Résumé—La stabilité de la convection dans une couche poreuse horizontale soumise à un gradient de température fini et confinée entre deux plans parfaitement conducteurs, est étudiée au moyen d'une analyse linéaire de stabilité. On montre que l'instabilité apparait sous la forme de rouleaux longitudinaux stationnaires (avec les axes alignés dans la direction de la composante horizontale du gradient de température) superposés à l'écoulement de base. Quand le nombre de Rayleigh horizontal augmente, le nombre de Rayleigh vertical critique croît aussi et il y a une série de transitions vers les modes plus élevés qui correspondent à des couches multiples de rouleaux.

KONVEKTION IN EINEM PORÖSEN MEDIUM MIT GENEIGTEN TEMPERATURGRADIENTEN

Zusammenfassung—Die Stabilität der Konvektion in einer waagerechten porösen Schicht wird mit Hilfe der linearen Stabilitätsanalysis untersucht, und zwar für den Fall eines geneigten endlichen Temperaturgradienten sowie ideal leitenden Begrenzungsflächen. Es zeigt sich, daß die Instabilität in Form stationärer Längswirbel auftritt, die der Grundströmung überlagert sind. Die Achsen der Wirbel sind in Richtung der horizontalen Komponente des Temperaturgradienten ausgerichtet. Bei wachsender horizontaler Rayleigh-Zahl nimmt die kritische senkrechte Rayleigh-Zahl ebenfalls zu, und es kommt zu einer Reihe von Übergängen zu Strömungsmustern höherer Ordnung, d.h. zu einer Vielzahl von Wirbellagen.

КОНВЕКЦИЯ В ПОРИСТОЙ СРЕДЕ С НАКЛОННЫМ ТЕМПЕРАТУРНЫМ ГРАДИЕНТОМ

Аннотация—С использованием линейного анализа исследуется устойчивость конвекции в горизонтальном пористом слое, подверженном действию наклонного температурного градиента конечной величины и ограниченного идеально проводящими плоскостями. Показано, что неустойчивость возникает в виде стационарных продольных валов (оси которых расположены в направлении горизонтального компонента температурного градиента), наложенных на основное течение. С увеличением горизонтального числа Рэлея возрастает критическое вертикальное число Рэлея и наблюдается ряд переходов к режимам более высокого порядка, соответствующим многочислокным слоям валов.